AQA Computer Science A-Level

4.4.5 A model of computation

Advanced Notes

Specification:

4.4.5.1 Turing machine:

Be familiar with the structure and use of Turing machines that perform simple computations.

Know that a Turing machine can be viewed as a computer with a single fixed program, expressed using:

- A finite set of states in a state transition diagram
- A finite alphabet of symbols
- An infinite tape with marked-off squares
- A sensing read-write head that can travel along the tape, one square at a time.

One of the states is called a start state and states that have no outgoing transitions are called halting states.

Understand the equivalence between a transition function and a state transition diagram.

Be able to:

- Represent transition rules using a transition function
- Represent transition rules using a state transition diagram
- Hand-trace simple Turing machines

Be able to explain the importance of Turing machines and the Universal Turing machine to the subject of computation.

Turing Machines

A Turing Machine is a formal model of computation that consists of a finite state machine, a read/write head and a tape that is infinitely long in one direction.

The tape is divided into cells, each of which can be left blank or contain a symbol. Symbols are written to and removed from cells on the tape by the Turing machine's read/write head. The set of symbols that a Turing machine uses is called its alphabet and must be finite.

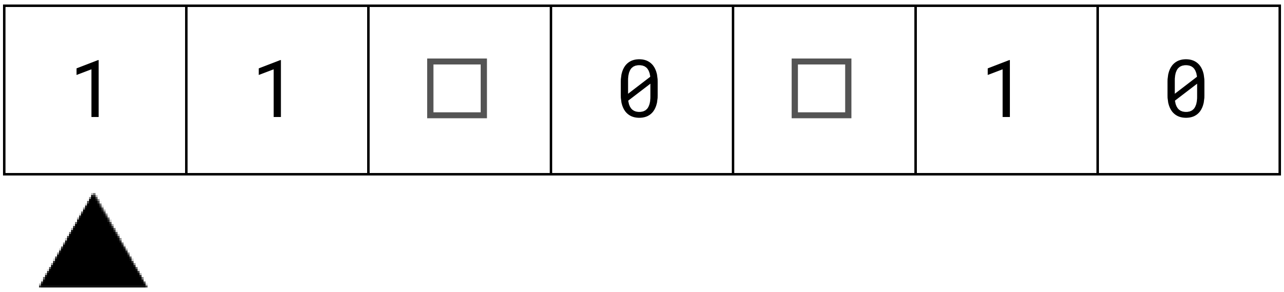
Synoptic Link

Finite state machines are covered under **regular languages**.

It's worth reading the notes on **finite state machines** before reading these notes.

A Turing machine can be viewed as a computer which runs a single program, as defined by a finite state machine. The finite state machine will have a start state and may have a number of states from which there are no transitions, referred to as halting states.

Turing machines stop after reaching their halting state. This state can be entered at any point in the machine's execution of its input data and is entered once all of the input data has been processed.



A Turing machine can be represented graphically as a series of cells, each containing a symbol, and a triangular pointer which represents the position of the machine's read/write head. A □ symbol signifies an empty cell.

As a model of computation, Turing machines are more powerful than finite state machines. This is because they can utilise a greater range of languages than finite state machines and because their tape is infinitely long in one direction.

Transition Functions

The rules that a Turing machine follows can be laid out using transition functions. These are written in the form:

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δ(current state, read) = (new state, write, move)
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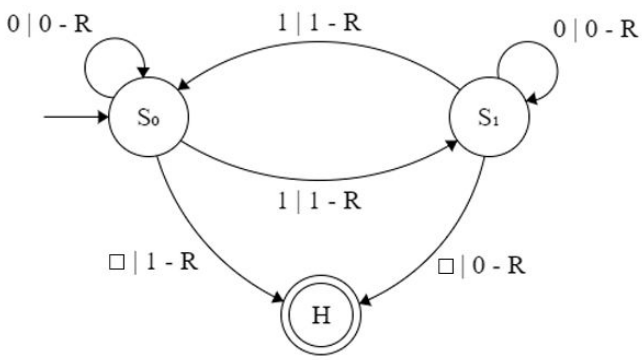
where δ is the Greek letter delta. For example:

$$\delta\ (S_0,\ \square) = (S_1,\ 1,\ R)$$

means if the machine is in state S_0 and reads an empty cell, the machine should write a 1, move to state S_1 and move its read/write head to the right.

There is an equivalence between transition functions and transition rules in a state transition diagram.



For example, the transition between state S_0 and state S_1 in the state transition diagram above could be written as a transition function like so:

$$\delta\ (S_0,\ 1) = (S_1,\ 1,\ R)$$

Example - Tracing a Turing machine

This example uses a Turing machine following the finite state machine in the state transition diagram above. Starting in S_0 (the start state) and with the data 110 on the tape.

S_0	1	1	0	□	□	□
	▲					

The Turing machine is in state S_0 and the read/write head reads a 1. In accordance with the state transition diagram, the Turing machine writes a 1, changes to state S_1 and moves to the right.

S_1	1	1	0	□	□	□
		▲				

Now in state S_1, the Turing machine reads a 1. As specified by the state transition diagram, the machine writes a 1 to the tape, moves to the right and changes to state S_0.

S_0	1	1	0	□	□	□
			▲			

The Turing machine is in state S_0 and reads a 0, so writes a 0, moves to the right and remains in state S_0.

S_0	1	1	0	□	□	□
				▲		

Now the Turing machine reads an empty cell and is in state S_0. The machine writes a 1, moves to the right and changes to state H, the halting state.

H	1	1	0	1	□	□
					▲	

Note

This Turing machine has applied odd parity to the data on its tape.

Universal Turing Machines

Turing machines are limited to following just one finite state machine, making them specific to the computational problem they need to solve.

Universal Turing machines are capable of representing any finite state machine. A description of the finite state machine to be used is read off of the same tape as the input data and then used to process the input data as usual.

> **Synoptic Link**
>
> Universal Turing machines are an example of the **stored program concept.**
>
> The stored program concept is covered in more detail under **computer organisation and architecture.**

Universal Turing machines can be said to act as interpreters because of the way they read their instructions in sequence before executing operations on their input data.

The importance of Turing machines

Turing machines provide a formal model of computation and therefore a definition of what is computable. The real importance of this to the subject of computation is that Turing machines can prove that there are problems which cannot be solved by computers.